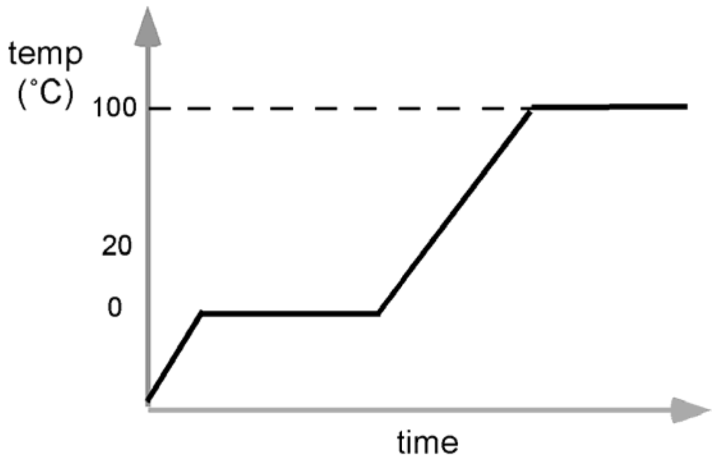
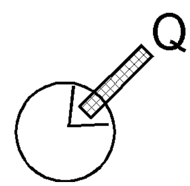
Unit 3 – Notes on Heating/Cooling Curve and Quantitative Energy Problems

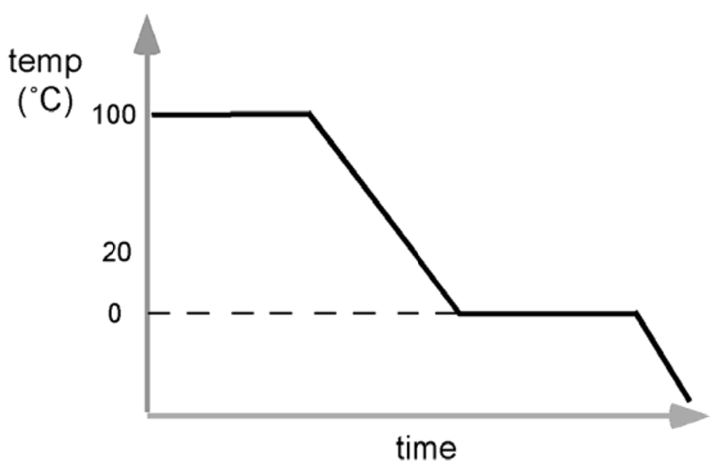
If you started with a sample of solid water well below the freezing point and supplied energy to it at a steady rate until it had partially boiled away, you would obtain a heating curve like the one below:



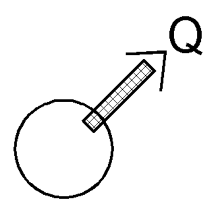
In our energy flow diagram we would show energy entering the system via heating during this series of changes. On the plateaus, the phase was changing and the system was storing E_{ph}. On the inclines, the temperature was changing and the system stored E_{th}.



If we had started with boiling water and allowed it to cool until it had frozen completely and cooled to below 0°C, we would have obtain a graph like the one below:



In our energy flow diagram we would show energy leaving the system via heating. On the plateaus, the system was giving up E_{ph} as the phase changed. On the declines, the temperature was changing and the system lost E_{th}.



We are now interested in learning just *how much* energy is transferred during these changes. From experiment, we have learned that it takes 4.18 joules[1] to raise the temperature of 1 g of liquid water by 1 °C. This amount of energy is equivalent to one calorie. We can write this value as a factor $\frac{4.18\,J}{1g \cdot 1°C}$. Suppose that we have a larger sample of liquid water, say 250 g. Clearly, it would take 250x as much energy to raise the temperature by one °C. In like manner, it would take 40x as much energy to raise the temperature by 40°C. We can show this in an equation: $Q = mc\Delta T$ where Q is the quantity of heat transferred, m represents the mass (in g), c is a property of liquid water known as the heat capacity, and ΔT is the change in temperature. Using the values above, $Q = 250g \cdot \frac{4.18J}{g°C} \cdot 40°C = 41{,}800J$ or $41.8kJ$. We usually use kJ as the unit for our answers because the joule is a pretty small unit of energy.

We note from experiment that ice, $H_2O(s)$, warms more rapidly than liquid water. Its heat capacity is $\frac{2.1J}{g°C}$. This means that only about half as much energy is required to raise the temperature of one gram of ice by one degree Celsius. Substances like metals have much lower heat capacities. You certainly have had experience with this fact if you have ever picked up a piece of metal that was lying in the sun. The radiant energy R, raised the temperature of the metal to an uncomfortably hot temperature.

We cannot use this relationship on the plateau portion of the heating (or cooling) curve because there the temperature is constant ($\Delta T = 0$). So we must use a different equation: $Q = m\Delta H_f$ when the substance is melting (or freezing) and $Q = m\Delta H_v$ when the substance is vaporizing (or condensing). Note that the quantity of energy is related to the mass of the substance times a property of that substance. For water, ΔH_f is 334 J/g, and ΔH_v is 2260 J/g. These values make sense when you consider that pulling apart molecules of liquid water until they become widely separated in a gas is more difficult than simply giving the solid water enough energy to allow the molecules to move freely past one another.

Calculations of energy changes on the plateaus are easy, but you have to make sure that you use the correct value of ΔH. To melt 50 g of ice requires $Q = 50g \cdot 334\, {}^{J}\!/_{g} = 16.7kJ$, but to vaporize that same quantity of water requires $Q = 50g \cdot 2260\, {}^{J}\!/_{g} = 113kJ$, a much greater amount.

[1] A joule is the SI unit of energy.